First Asia-Pacific Solar Physics Meeting
ASI Conference Series, 2011, Vol. 2, pp 221–227
Edited by Arnab Rai Choudhuri & Dipankar Banerjee

Alfvén waves are easy: mode conversion in magnetic regions

P. S. Cally[1,2*]
[1]*Monash Centre for Astrophysics and School of Mathematical Sciences, Monash University, Clayton 3800, Victoria, Australia*
[2]*High Altitude Observatory, National Center for Atmospheric Research, P.O. Box 3000, Boulder, CO 80307, USA*[†]

Abstract. Alfvén waves are shown to be readily generated by mode conversion from fast MHD waves reflecting off the steep atmospheric Alfvén speed gradient in active region atmospheres. A simple analytic description of this process in terms of an 'interaction integral' indicates that it is spread over many vertical scale heights, and indeed fills the whole active region chromosphere for waves of moderate helioseismic degree ℓ, even up to $\ell = 1000$ or more. This suggests that active region chromospheres are Alfvén wave factories.

Keywords : Sun: oscillations – MHD – waves

1. Introduction

Alfvén (1942) first described the incompressive magnetohydrodynamic waves that now bear his name, and postulated that they may have implications for sunspots. Certainly, Alfvén waves have long been detected *in situ* in the solar wind (Belcher & Davis 1971), where they may take the form of outgoing large amplitude shear waves, typically in fast streams (Velli & Pruneti 1997) and especially out of the ecliptic, or drive interstellar turbulence by nonlinear interaction (Goldreich & Sridhar 1995).

The dominance of Alfvén waves travelling away from the Sun in high speed streams naturally suggests a solar origin. In recent years transverse (to the magnetic field) waves have been shown to be ubiquitous throughout the corona (De Pontieu et

*email: paul.cally@monash.edu
†The National Center for Atmospheric Research is sponsored by the National Science Foundation.

al. 2007; Tomczyk et al. 2007), though there is ongoing argument about whether they are Alfvén waves or kink waves (Van Doorsselaere, Nakariakov & Verwichte 2008). The two (related) wave types can apparently couple quite readily though in flux tubes with transverse structure (Pascoe, Wright & De Moortel 2010, 2011), and may share a common origin irrespective of which is actually responsible for the observed coronal oscillations.

The natural instinct is to suspect direct side-to-side shaking by convection at the photosphere as the excitation mechanism, but this may be shown to be inefficient (Collins 1992; Parker 1991). Unresolved transition region nano-flare excitation has also been suggested (Parker 1991; Velli & Pruneti 1997). The propagation of Alfvén waves through the many scale heights of the photosphere-chromosphere and into the corona and beyond has been widely modelled (e.g., Orlando et al. 1996; Cranmer, van Ballegooijen & Edgar 2007) in many magnetic scenarios.

But what of other wave types? As pointed out by Velli & Pruneti (1997), the slow wave is essentially acoustic in a low β plasma, and is subject to reflection if the frequency is below the acoustic cutoff. It should be recalled though that inclined magnetic field reduces the effective cutoff frequency, the so-called ramp effect, to open up 'magnetoacoustic portals' to the upper atmosphere (Bel & Leroy 1977; Schwartz, Cally & Bel 1984; Jefferies et al. 2006). Sound waves are also subject to nonlinear steepening and shocking as the density decreases with height through the chromosphere. On the other hand, fast waves, which are predominantly magnetic in character, totally refract from the steep Alfvén speed gradient (Schunker & Cally 2006; Cally 2007; Nutto, Steiner & Roth 2010). As explained by Schunker & Cally (2006), both these fast and slow waves can be atmospheric extensions of the Sun's internal p-mode wave field that undergoes transmission and conversion near the Alfvén-acoustic equipartition surface in regions of high magnetic field, sunspots in particular. Strong surface field thereby opens an 'escape hatch' for the p-mode power normally trapped beneath the photosphere.

However, this is not the full story. Although the fast wave reflects (roughly at the height where $\omega = a\,k$, with a the Alfvén speed and k the horizontal wavenumber), it couples to the third MHD wave type, the Alfvén wave, provided that gravity $\mathbf{g}$, the magnetic field $\mathbf{B}$, and the wavevector $\mathbf{k}$ are not coplanar (Cally & Goossens 2008). This scenario has been investigated recently by Cally & Hansen (2011) for the simplest model possessing the necessary features: a zero β plasma with exponentially increasing Alfvén speed, and uniform magnetic field oriented at angle θ to the direction of inhomogeneity. Because the interest is chiefly in the fast/Alfvén coupling, the slow wave is frozen out with the $\beta = 0$ assumption.

In this paper, we briefly describe the results of this model, with particular attention on where the coupling occurs (sketching a new analysis), and discuss implications for Alfvén waves in active region atmospheres.

2. Where does fast-to-Alfvén conversion occur?

Although fast waves are evanescent above their classical reflection points, their fall-off with height in a roughly isothermal layer such as the solar chromosphere has exponential controlling factor $\exp(-k\,z)$. For helioseismic waves of moderate order $\ell \sim 200$ say (as an example) we have $k \sim 0.3\ \mathrm{Mm}^{-1}$, suggesting their influence will extend throughout the chromosphere (roughly 2 Mm thick). Consequently, mode coupling between the fast and Alfvén waves occurs precisely in this distended evanescent tail. The main purpose if this paper is to justify this contention.

Mode conversion quite generally occurs where the two waves have nearly equal phase velocities (and compatible polarizations), so that their wave trains fit together like gear cogs. Mathematically, this is normally manifested as a stationary phase integral. This is certainly the case for fast-slow conversion (Cally 2005). Turning now to the fast-Alfvén case, and neglecting the sound speed c in comparison to the Alfvén speed a, the fast wave has dispersion relation $\omega^2 = a^2|\mathbf{k}|^2$ and the Alfvén wave is described by $\omega^2 = a^2 k_\parallel^2$, where ω is frequency, $\mathbf{k}$ is the wavevector, and $k_\parallel$ is the component of $\mathbf{k}$ in the direction of the magnetic field. However, this WKB description, although suggestive, may not be strictly valid in the fast wave's evanescent tail where the (imaginary) vertical wavenumber, $k_z \approx i\,k$, may be comparable to the inverse scale height h^{-1}. Indeed, $kh \ll 1$ is more relevant to p-mode driving, as discussed below.

A more exact treatment is possible. The governing equation describing fast-Alfvén interaction is

$$\left(\partial_\parallel^2 + \frac{\omega^2}{a^2}\right)\boldsymbol{\xi} = -\boldsymbol{\nabla}_p \chi\,, \tag{1}$$

where $\chi = \boldsymbol{\nabla}\boldsymbol{\cdot}\boldsymbol{\xi}$ is the dilatation, $\boldsymbol{\xi}$ is the plasma displacement, $\partial_\parallel = \hat{\mathbf{B}}\boldsymbol{\cdot}\boldsymbol{\nabla}$ is the field-aligned derivative, and $\boldsymbol{\nabla}_p$ is the part of the gradient perpendicular to the magnetic field. The displacement $\boldsymbol{\xi} = \boldsymbol{\xi}_p$ is entirely transverse to the field lines. An $\exp[-i\omega t]$ time dependence is assumed throughout. In this form we see that χ, representing the (compressive) fast wave, acts as a source term for Alfvénic disturbances on field lines. Only where the source is resonant with the Alfvén wave is there significant interaction.

Let us assume a steady 'source' $\chi = \chi_0$, and define X_+ and X_- to be the solutions of the homogeneous Alfvén equation $\left(\partial_\parallel^2 + \omega^2/a^2\right)X = 0$ representing waves propagating in the direction of increasing and decreasing Alfvén speed respectively. The solution of the full driven equation is then

$$\boldsymbol{\xi} = \mathbf{a}(\sigma)\,X_-(\sigma) + \mathbf{b}(\sigma)\,X_+(\sigma) \tag{2}$$

where

$$\mathbf{a}(\sigma) = \int_\sigma^\infty W^{-1} X_+(\sigma')\,\boldsymbol{\nabla}_p\chi_0(\sigma')\,d\sigma'\,, \quad \mathbf{b}(\sigma) = \int_{-\infty}^\sigma W^{-1} X_-(\sigma')\,\boldsymbol{\nabla}_p\chi_0(\sigma')\,d\sigma' \tag{3}$$

and σ is distance along a field line. The Wronskian of the two linearly independent

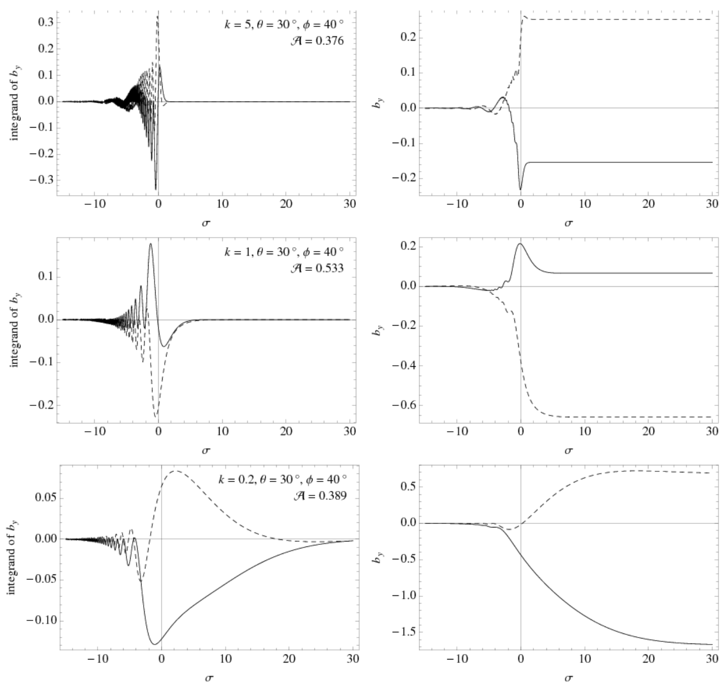

Figure 1. Integrand appearing in b_y (left column) and $b_y(\sigma)$ itself (right column) for $k = 5$, 1, and 0.2 (top to bottom) for the case $\theta = 30°$, $\phi = \arctan(k_y/k_x) = 40°$. The vertical line at $\sigma = 0$ indicates the classical position of fast wave reflection. The Alfvén conversion coefficient $\mathcal{A}$ of each case is indicated in the left panels.

solutions, $W = X_+X'_- - X'_+X_-$, is necessarily constant for a second order equation without first derivative.

To be specific, and in line with Cally & Hansen (2011), let a^2 increase exponentially with height z, with the density scale height arbitrarily set to unity.[1] With horizontal wavenumber $k = \sqrt{k_x^2 + k_y^2}$, the uncoupled χ solution is

$$\chi_0 = J_{2k\sec\theta}\left(2e^{-z/2}k\sec\theta\right) e^{i(k_x x + k_y y)}\,, \tag{4}$$

in terms of the Bessel function of the first kind. This may be regarded as the first term in a perturbation expansion; later terms will see the reflected fast wave component reduced in magnitude due to Alfvén leakage. Then, with magnetic field oriented at angle θ from the vertical and lying in the x-z plane,

$$X_\pm = H_0^{(2,1)}\left(2e^{-(\sigma/2)\cos\theta}k\sec\theta\right) \tag{5}$$

[1]Cally & Hansen take this direction of inhomogeneity to be x, but in light of the context here, we label it z.

on the typical field line passing through $x = 0$, $y = 0$, $z = 0$, so that $x = \sigma \sin\theta$, $y = 0$, and $z = \sigma\cos\theta$. $H_0^{(1)}$ and $H_0^{(2)}$ are the Hankel functions of order zero of the first and second kind respectively. The Wronskian satisfies $W^{-1} = i(\pi/2)\sec\theta$. Without loss of generality, the fast mode turning point has been placed at $z = 0$. We term **a** and **b** the 'interaction integrals'. They represent the 'turning on' of the downgoing and upgoing Alfvén waves respectively.

For simplicity, we concentrate on ξ_y (this is not quite the correct polarization of the Alfvén wave, see Cally & Hansen (2011) for details, but it will suffice to illustrate the nature of the interaction integrals). Both the integrand of b_y and $b_y(\sigma)$ itself are plotted in Fig. 1 for a representative case: $\theta = 30°$, $\phi = \arctan(k_y/k_x) = 40°$. Remember that the vertical density scale height h has been set to 1. With $k = 5$ (top row) the Alfvén wave turns on sharply around the fast wave reflection point, indicating a very compact fast-to-Alfven conversion region because the fast wave evanescent tail is very short. For $k = 1$ (centre row) conversion is spread over several scale heights. Finally, for $k = 0.2$ (bottom) the interaction of the two wave types is spread over 20 or more scale heights, easily encompassing the whole chromosphere. To put this in context, for a density scale height of 150 km dimensionless $k = 5$ corresponds to a dimensional $k = 1.33$ Mm^{-1}, or $\ell = 928$. This is definitely in the high-ℓ region. Conversion of helioseismic waves of more moderate ℓ is even more spread in height. Note in Fig. 1 how highly oscillatory behaviour of the integrand below the reflection point $z = 0$ largely cancels, leaving the evanescent tail as the major source of Alfvén wave excitation at small to moderate k.

Numerically, the fast-to-Alfvén conversion coefficients $\mathcal{A}$ (fraction of energy converted to the outgoing Alfvén wave) for the three case used as illustration are: 0.376 for $k = 5$; 0.533 for $k = 1$, and 0.389 for $k = 0.2$. The reader is referred to Cally & Hansen (2011) for a detailed survey of absorption coefficients as a function of k, θ, and ϕ. Suffice it to say here that they can be quite substantial.

3. Discussion

Helioseismic waves emerging in strong surface magnetic field concentrations such as sunspots are known to split into a slow wave (acoustic) and a fast wave (magnetic) near the Alfvén acoustic equipartition level. The fast wave then reflects off the imposing Alfvén speed gradient found in the low solar atmosphere due to the small density scale height. Provided the magnetic field, gravity, and the wavevector are not co-planar, there is then further conversion from the fast wave to an Alfvén wave. This occurs at and beyond the fast wave reflection height, and is spread over many scale heights for wavenumbers typical of local helioseismology.

In particular, we postulate that the whole active region chromosphere must be an 'Alfvén wave excitation layer'. We might expect similar behaviour in smaller scale flux concentrations such as network, though we have not modelled this (in such elements, cross-field inhomogeneities may dominate though). Of course, there will be reflection at the transition region. However, this generation of transverse waves below the corona may be a potent source of transverse (Alfvén or kink) oscillations observed throughout the solar corona. This suggestion is supported by the observed excess power in coronal Alfvénic oscillations around 5 minutes (Tomczyk et al. 2007, Fig. 2), indicating a link to p-modes (see also the early identification of 5 min periodicities in OSO8 data throughout the chromosphere and into the transition region, White & Athay 1979; Athay & White 1979).

Finally, it should be pointed out that conversion to the *downgoing* Alfvén wave through the other interaction integral, $\mathbf{a}(\sigma)$, is favoured where $\theta > 90^\circ$ (or equivalently $0 < \theta < 90^\circ$ but $k_x < 0$). This is discussed in detail in Cally & Hansen (2011). Essentially, if the fast wave draws roughly parallel to the magnetic field on its 'upstroke' (prior to reflection), it converts to an upgoing Alfvén wave, but if the directional correspondence occurs on the downstroke, the downgoing Alfvén wave results. The latter may be expected to have no implications for the corona (unless further reflection is important). Hence, magnetic field geometry can act as a filter on Alfvén waves produced in the low solar atmosphere.

Acknowledgements

The author thanks Phil Judge for pointing out some references and suggesting novel insights.

References

Alfvén H., 1942, Nature, 150, 405
Athay R. G., White O. R., 1979, ApJ, 229, 1147
Bel N., Leroy B., 1977, A&A, 55, 239
Belcher J. W., Davis L., Jr., 1971, JGR, 76, 3534
Cally P. S., 2005, MNRAS, 358, 353
Cally P. S., 2007, AN, 328, 286
Cally P. S., Goossens M., 2008, Solar Phys., 251, 251
Cally P. S., Hansen S. C., 2011, ApJ, 738, 119
Collins W., 1992, ApJ, 384, 319
Cranmer S. R., van Ballegooijen A. A., Edgar R. J., 2007, ApJS, 171, 520
De Pontieu B., et al., 2007, Sci., 318, 1574
Goldreich P., Sridhar S., 1995, ApJ, 438, 763
Jefferies S. M., McIntosh S. W., Armstrong J. D., Bogdan T. J., Cacciani A., Fleck B., 2006, ApJ, 648, L151
Nutto C., Steiner O., Roth M., 2010, AN, 331, 915
Orlando S., Lou Y.-Q., Rosner R., Peres G., 1996, JGR, 101, 24443
Parker E. N., 1991, ApJ, 372, 719

Pascoe D. J., Wright A. N., De Moortel I., 2010, ApJ, 711, 990
Pascoe D. J., Wright A. N., De Moortel I., 2011, ApJ, 731, 73
Schunker H., Cally P. S., 2006, MNRAS, 372, 551
Schwartz S. J., Cally P. S., Bel N., 1984, Solar Phys., 92, 81
Tomczyk S., McIntosh S. W., Keil S. L., Judge P. G., Schad T., Seeley D. H., Edmondson J., 2007, Sci., 317, 1192
Van Doorsselaere T., Nakariakov V. M., Verwichte E., 2008, ApJ, 676, L73
Velli M., Pruneti F., 1997, PPCF, 39, 317
White O. R., Athay R. G., 1979, ApJS, 39, 347