



## Glitch statistics of radio pulsars: Multiple populations

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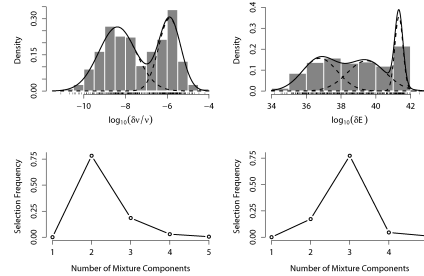
**Abstract.** We present statistical evidence suggesting more than one population in the energy distribution of pulsar glitches, which implies the presence of different mechanisms accessing different energy ranges responsible for glitches.

A glitch is a timing irregularity of radio pulsars, marked by a sudden increase in the spin-frequency  $\nu$ , often followed by a relaxation towards the unperturbed  $\nu$ . A total of 438 glitches have been seen in 150 pulsars so far (Manchester et al. 2005; Espinoza et al. 2011). These are likely caused by sudden and irregular transfer of angular momentum to the solid crust of the neutron star by a super-fluid component rotating faster; or by the crust quakes. It is conjectured that the bimodality seen within the range of glitch values ( $10^{-12} \leq \delta\nu/\nu \leq 10^{-4}$ ) are indicative of these two separate mechanisms (Yu et al. 2013). As a step towards understanding the mechanisms underlying glitches, we consider the statistical nature of the glitch energy ( $E_g$ ) distribution. The rotational energy,  $E_r$ , of a pulsar is approximately  $I\nu^2$ , where  $I$  is the stellar moment of inertia. The change in rotational energy due to a glitch is then  $E_g = \delta E_r \approx I\nu\delta\nu$ , assuming  $I$  to be roughly constant ( $\approx 10^{45}$  gm.cm<sup>2</sup>) across the glitching pulsar population.

Multimodality of distribution of  $\log_{10}(\delta\nu/\nu)$  and  $\log_{10}(\delta E)$  data would suggest the presence of multiple populations. We therefore apply three standard statistical tests; namely, dip test (Hartigan & Hartigan 1985), Silverman test (Silverman 1981), and bimodality test (Holzmann & Vollmer 2008) for which the null hypothesis is that of unimodality. As a measure of evidence in favour of the null hypothesis, we report the  $p$ -value for each test ( $0 \leq p \leq 1$ ; lower the  $p$ , greater the evidence against the null hypothesis). We also apply a multimodel bootstrap approach coupled with BIC-based model selection (Burnham & Anderson 2002) to obtain model selection frequencies for Gaussian mixtures (McLachlan & Peel 2000) with 1–5 components fitted to the glitch energy data. Here, the selection frequency for the 1-component model is akin to the  $p$ -value for a test. The top row in Fig.1 shows data histograms

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**Figure 1.** Statistical evidence in favour of more than one pulsar population; see text for description.

together with BIC-optimal Gaussian mixture fits. For the  $\log_{10}(\delta\nu/\nu)$  data, the  $p$ -values are very small ( $\approx 10^{-4}$ , 0, and  $10^{-11}$  respectively), strongly rejecting the null hypothesis of unimodality. For the  $\log_{10}(\delta E)$  data, the  $p$ -values are  $\approx 0.12$ , 0, and  $10^{-14}$  respectively. The dip test  $p$ -value of 0.12, though closer to 0 than to 1, suggests only weak evidence against unimodality. However, a combination of the three  $p$ -values into a single one (Vovk 2012) suggests moderate evidence against unimodality in the  $\log_{10}(\delta E)$  data as well. The bottom row shows model selection frequencies as functions of the number of mixture components (bootstrap size: 10000), where we clearly see that the 1-component model is the least favoured model for either data set. All results taken together suggest moderate statistical evidence for multimodal structure in the data, suggestive of more than one glitch mechanisms corresponding to different intrinsic energies.

This purely agnostic statistical analysis also seems to have some grounding in the reality: e.g., all known magnetar glitches in the  $\log_{10}(\delta E)$  data fall under the rightmost Gaussian, the left hand edge of which is around  $E_g \sim 10^{40}$  gm.cm<sup>2</sup>.sec<sup>-1</sup>. This is very close to the maximum energy available to the outer crust of a neutron star (Mandal et al. 2009), implying that the higher energy glitches would have to come from deeper regions of the crust. Andersson *et al.* (2011) have also concluded that crustal energy budget may not be sufficient to explain every glitch. This implies that mechanisms responsible for glitches may be different for different energy ranges.

## References

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