The Metrewavelength Sky
ASI Conference Series, 2014, Vol. 13, pp 393 – 394
Edited by J. N. Chengalur & Y. Gupta

Non-linear redundancy calibration

Visweshwar Ram Marthi* and Jayaram Chengalur
National Centre for Radio Astrophysics, Tata Institute of Fundamental Research, Ganeshkhind, Pune 411007, India

Abstract. A steepest descent based algorithm to obtain the gains of the antennas and the model visibilities of the observed sky, exploiting a redundant array geometry, is described, as relevant to the upgrade of the Ooty Radio Telescope. Simulations establish that the estimator is statistically efficient.

Keywords : methods: numerical – methods: statistical – techniques: interferometric

1. Redundancy calibration

A radio telescope with regular spacing between the antennas produces multiple instances of the same baseline vector. The radio interferometer measurement equation

$$V_{ij} = g_i g_j^* M_{ij} + N_{ij} \tag{1}$$

becomes an overdetermined system of equations. The complex antenna gains as well as the visibilities can be solved for simultaneously: this is the defining feature of redundancy calibration. These non-linear equations are linearized by taking logarithm(Wieringa 1991, 1992) or by expanding as a Taylor series(Liu et al. 2010). The logarithmic method produces biased solutions except at high SNR. On the other hand, the Taylor series method is expensive - computations go as N^4. These two methods are classified under the Linear Least Squares(LLS) minimization method. LLS methods solve for amplitudes and phases, leading to quadrant ambiguity of estimated phases.

*email: vrmarthi@ncra.tifr.res.in

2. Non-linear redundancy calibration

We define a real-valued objective function to be minimized with respect to the gains g_i and model visibilities $M_{|i-j|}$:

$$\Lambda = \sum_i \sum_{j>i} w_{ij} || \left(V_{ij} - g_i g_j^* M_{|i-j|} \right) \left(V_{ij}^* - g_i^* g_j M_{|i-j|}^* \right) || \quad (2)$$

The solutions are

$$\mathbf{Q_k} = \frac{\sum_{j \neq k} w_{kj} g_j M_{|k-j|}^* V_{kj}}{\sum_{j \neq k} w_{kj} |g_j|^2 \, |M_{|k-j|}|^2} \quad \& \quad \mathbf{R_{kj}} = \frac{\sum_{j>k} g_k^* g_j V_{kj}}{\sum_{j>k} w_{kj} |g_k|^2 |g_j|^2} \quad (3)$$

which can be iteratively refined as

$$g_k^{n+1} = (1-\alpha) g_k^n + \alpha \mathbf{Q_k}^n \quad \& \quad M_{|k-j|}^{n+1} = (1-\alpha) M_{|k-j|}^n + \alpha \mathbf{R_{kj}}^n \quad (4)$$

It can be seen that the complexity goes as N^2, and is therefore faster than LLS methods. Moreover, the complex solutions are rectangular instead of polar. Similar to self-calibration, amplitudes and phases are to be fixed only by external calibration. Besides, the estimator is found to be statistically efficient, as it achieves the Cramér-Rao bound(CRB).

Since the Ooty Radio Telescope(ORT) is undergoing an upgrade(see Prasad and Subrahmanya, 2011) that enables it as an interferometer with a highly redundant geometry, this calibration strategy finds direct application. Besides, among many other post-calibration uses, the calibrated visibilities can be combined coherently to form beams on the sky towards well-targeted co-ordinates.

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