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Parity of solar global magnetic field determined by turbulent diffusivity

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Abstract. We investigate the criterion for the solar dipole-field in a kinematic flux-transport dynamo model. The sun has a dipole-like global magnetic field. This field is thought to be generated by the dynamo action of the solar internal plasma. The flux-transport dynamo succeeds to reproduce some features of solar cycle, e.g. poleward the migration of the general magnetic field and the butterfly diagram. The parity, however, of the global magnetic field significantly depends on parameters in the flux-transport dynamo. It is known that the coupling of the magnetic field between hemispheres due to turbulent diffusivity is an important factor for the solar parity issue, but the detailed criterion for the generation of the dipole field has not been investigated. Our conclusions are as follows. (1) The stronger diffusivity near the surface is more likely to cause the magnetic field to be a dipole. (2) The thinner layer of the strong diffusivity near the surface is also more apt to generate a dipolar magnetic field. (3) The faster meridional flow is more prone to cause the magnetic field to be a quadrupole, i.e., symmetric about the equator. The result (1) is consistent with our previous work (Hotta & Yokoyama 2010a), which is on the effect of the surface diffusivity for the observed weak polar field.

Keywords : Sun: dynamo – Sun: activity

1. Introduction

The number of sunspots, i.e. the solar magnetic fields, has an eleven-year cyclic variation. This solar cycle is thought to be sustained by the dynamo action (Parker 1955). Dynamo means the transformation from the kinetic energy to the magnetic energy.

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The flux-transport dynamo can reproduce some features of the solar cycle, such as the poleward migration of the general magnetic field and the equatorward migration of the active latitude (Choudhuri, Shüssler & Dikpati 1995; Dikpati & Charbonneau 1999). The flux-transport dynamo consists of three steps. At the first step, strong toroidal magnetic fields are generated by the steep radial gradient of angular velocity at the tachocline (Ω-effect). Then the toroidal fields begin to rise to the surface due to the magnetic buoyancy. During the rising, the toroidal fields are bent by the Coriolis force and poloidal magnetic fields are generated near the surface (Babcock-Leighton α-effect). This is the second step. In the third step, the generated poloidal magnetic fields are transported to the tachocline again by the meridional flow or the turbulent diffusion from the surface to the tachocline. The toroidal magnetic field is generated again at the tachocline, i.e. the first step. An important thing of the flux-transport dynamo for this study is that the toroidal fields are generated mainly around the base of the convection zone, i.e. tachocline and the poloidal fields are generated near the surface.

The solar global magnetic field is antisymmetric about the equator, i.e. the dipolar global magnetic field. The north and south polar fields are almost always opposite and Hale's law states the structure of each sunspot pair is antisymmetric about the equator. In the flux-transport dynamo, however, the parity significantly depends on free parameters, i.e. the meridional flow and the turbulent diffusivity.

Chatterjee, Nandy & Choudhuri (2004) suggested that the turbulent diffusivity has an important role on the parity issue. This role can be explained with Fig 1. The panel (a) of Fig. 1 shows the poloidal field in the dipole and quadrupole cases. In the dipole case, the sign of the ϕ-component vector potential is same between hemispheres. If the phase in one hemisphere advances faster than the other hemisphere, this difference can be smoothed out with turbulent diffusion. On the other hand, this effect does not work in quadrupole case because the sign of the vector potentials are opposite. We conclude that the turbulent diffusion for the poloidal magnetic fields is likely to generate the global magnetic field dipole. The sign of the toroidal magnetic fields are opposite (same) between hemispheres in dipole (quadrupole) case. With the same logic as the case for the poloidal field, it is clear that the turbulent diffusion of the toroidal field is prone to generate quadrupole field.

In this study, we investigate the dependence of the parity of solar global magnetic field on the distribution of the turbulent diffusivity and the speed of the meridional flow.

2. Model

Axisymmetric magnetic induction equation is solved kinematically in spherical geometry (r, θ, ϕ), where r, θ and ϕ denote the radius, the colatitude and the longitude, respectively. The magnetic field is decomposed into the toroidal field B and the poloidal

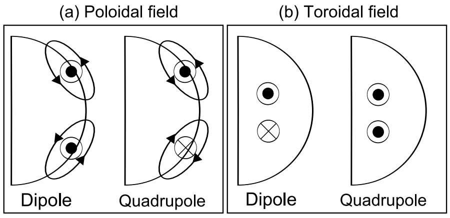

Figure 1. Illustration of the parity issue. Panel (a) shows the poloidal fields (line) for a dipole and a quadrupole field and the corresponding vector potentials. Panel (b) shows the toroidal field for a dipole and a quadrupole.

field $\mathbf{B_p} = \nabla \times (A\mathbf{e}_\phi)$, where A is the ϕ component of the magnetic vector potential and $\mathbf{e}_\phi$ is the unit vector along the ϕ-direction. We obtain equations for the toroidal and poloidal field as follows,

$$\frac{\partial B}{\partial t}+\frac{1}{r}\left[\frac{\partial}{\partial r}(ru_rB)+\frac{\partial}{\partial \theta}(u_\theta B)\right]=r\sin\theta(\mathbf{B_p}\cdot\nabla)\Omega-(\nabla\eta\times\nabla\times B\mathbf{e}_\phi)\cdot\mathbf{e}_\phi+\eta\left(\nabla^2-\frac{1}{r^2\sin^2\theta}\right)B, \tag{1}$$

$$\frac{\partial A}{\partial t}+\frac{1}{r\sin\theta}(\mathbf{u}\cdot\nabla)(r\sin\theta A)=\eta\left(\nabla^2-\frac{1}{r^2\sin^2\theta}\right)A+S(r,\theta;B). \tag{2}$$

The meridional flow $\mathbf{u} = (u_r, u_\theta)$, the differential rotation Ω, the turbulent diffusivity η and the source term of the Babcock-Leighton α-effect S are took as parameters. The detailed settings of the meridional flow, the differential rotation and the Babcock-Leighton α-effect are shown in Hotta & Yokoyama (2010b).

The turbulent diffusivity is defined as follows,

$$\eta(r)=\eta_{\rm core}+\frac{\eta_{\rm t}}{2}\left[1+\mathrm{erf}\left(\frac{r-r_1}{d_1}\right)\right]+\frac{\eta_{\rm s}}{2}\left[1+\mathrm{erf}\left(\frac{r-r_2}{d_2}\right)\right]. \tag{3}$$

Here $r_1 = 0.7R$, $d_1 = 0.02R$, and $d_2 = 0.02R$, where R is the solar radius. $\eta_{\rm s}$, $\eta_{\rm t}$, and $\eta_{\rm core}$ denote the values of the turbulent diffusivity near the surface, in the middle of convection zone and around the bottom of convection zone, respectively. We use the fixed value $\eta_{\rm t} = 5\times10^{10}$ cm^2 s^{-1} and $\eta_{\rm core} = 5\times10^8$ cm^2 s^{-1}. We take $\eta_{\rm s}$ and r_2 as free parameters. For convenience, we define the surface depth $d_{\rm s} = R - r_2$ which denotes the thickness of the strong diffusivity layer.

3. Result

A new indicator of the magnetic parity is defined in this study. The radial magnetic field at the surface can be decomposed with the Legendre polynomial P_n as,

$$B_r(R,\theta)=\sum_{n=0}c_nP_n(\cos\theta). \tag{4}$$

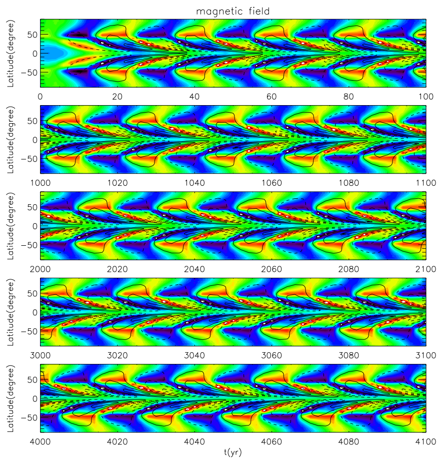

Figure 2. Butterfly diagram for the reference solution. The time-latitude plot of $B_\phi|_{r=0.7R}$ by contour is superposed on the color map of the surface radial fields.

Then, we define the symmetric parameter (SP) with the ratio of the coefficient of even and odd order of the Legendre polynomial as

$$\mathrm{SP} = \frac{\sum_{i=0} |c_{2i}| - \sum_{i=0} |c_{2i+1}|}{\sum_{i=0} |c_i|}. \tag{5}$$

Each even (odd) order of the Legendre polynomial is symmetric (antisymmetric) about the equator. When SP = 1 (SP = −1) the global magnetic field is completely symmetric (antisymmetric) about the equator.

Fig. 2 shows the time-latitude plot of the result with our reference parameter. The amplitude and the thickness of the turbulent diffusivity near surface is $\eta_\mathrm{s} = 2 \times 10^{12}$ cm^2 s^{-1} and $d_\mathrm{s} = 0.1R$, respectively. This simulation starts with a symmetric initial condition. As time goes on, the global magnetic field becomes antisymmetric about the equator. In this case, the SP develops as shown in Figure 3. The black (red) line denotes the result with a symmetric (antisymmetric) initial condition. Regardless of the initial condition, the value of SP becomes about −1 i.e. the antisymmetric value.

We investigate the asymptotic stationary values of the SP for runs with different free parameters. We carried out runs by choosing a value for η_s, from eight points in the range 6×10^{11} cm^2 s^{-1} to 1×10^{13} cm^2 s^{-1} and d_s, from five points in the range 0.1-0.25R. The speed of the meridional flow u_0 is also took as a free parameter and 1000 cm s^{-1} (slow case) and 2000 cm s^{-1} (fast case) are adopted. The results of this parameter-space study are shown in Fig. 4. Panel (a) shows the result of the slow meridional flow case. The contour plot of SP is shown. This result indicates two important points. One is that the strong turbulent diffusivity near the surface likely to generate a dipolar field. The other is that a thinner surface depth d_s is more prone to make the global magnetic field dipole. As we explained in §1, toroidal fields

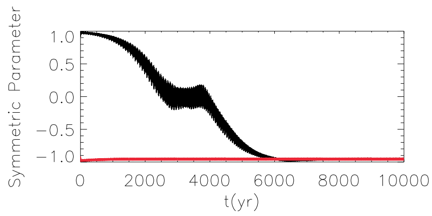

Figure 3. Time development of the SPs. The black (red) line corresponds to the results of the symmetric (antisymmetric) initial condition. Regardless of the initial condition, the SP finally becomes ~ -1 (antisymmetric solution).

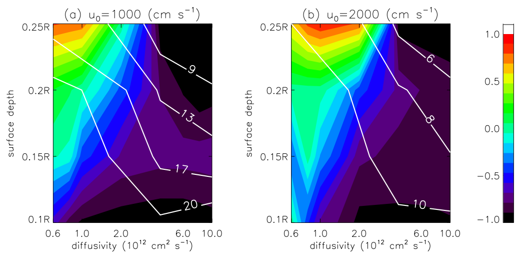

Figure 4. Symmetric parameter as a function of the diffusivity η_{s} and the surface depth ds. The superposed lines indicate the contours of the dynamo cycle period over periods of years. Panel (a) shows the results for the slow meridional flow case (u_0 = 1000 cm s^{-1}). Panel (b) shows the result for the fast meridional flow case (u_0 = 2000 cm s^{-1}).

are generated mainly around the tachocline and poloidal fields are generated near the surface. The strong turbulent diffusion near the surface mainly promotes the coupling of poloidal fields between the hemispheres. In addition, the coupling of toroidal fields is suppressed, when the surface depth becomes small. Both effects, i.e. strong and thin, make the global magnetic field dipole.

Panel (b) shows the result with the fast meridional flow. It is clear that the criterion for the global magnetic field to be a quadrupolar field becomes relaxed. In the flux-transport dynamo, the meridional flow transports poloidal fields poleward and toroidal fields equatorward. This effect suppresses the coupling of poloidal fields and promotes the coupling of toroidal fields. Both effects make the global magnetic field quadrupole.

4. Discussion and summary

We investigated the dependence of the global magnetic parity on the distribution of the diffusivity (the amplitude and the surface depth) and the amplitude of the meridional flow. Three results were obtained. First, the model shows that the stronger diffusivity near the surface acts to make the magnetic field a dipole. The diffusivity near the surface enhances mainly the coupling of the poloidal field near the surface between the hemispheres, leading to the generation of a dipolar magnetic field. The second result is that the thinner layer of the strong surface diffusivity also works to cause the magnetic field to become dipolar. The first result is consistent our previous study (Hotta & Yokoyama 2010a), which is on the effect of the surface diffusivity for the observed weak polar field. The thinner surface depth suppresses the coupling of the toroidal field between the hemispheres since most of the toroidal field exists around the tachocline. The third result is that the fast meridional flow causes the magnetic field to become a quadrupole. The fast meridional flow prevents the poloidal field from coupling near the surface of the equator because the flow transports the poloidal field poleward. In addition, the flow transports the toroidal field around the tachocline equatorward, thus causing the coupling of the toroidal field. These three results quantitatively constrain the distribution and the amplitude of turbulent diffusivity, which cannot be determined by observation and is an important factor for the dynamo problem.

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