Lecture 3 : Why are the stars as they are?

In the early part of the twentieth century, great intellectuals like Sir Arthur Eddington and (a very young) S. Chandrasekhar were able to explain some of the fundamental characteristics of stars from basic principles of physics. In fact, Sir Eddington demonstrated that it was possible to infer about the existence and nature of stars without ever knowing about them!

In the first lecture, stars were described as globes of gas in which the inward pull of gravity is balanced by the pressure of the gas. In this lecture, Eddington's essential modifications to this idea are described with the introduction of the pressure of the radiation trapped in the star. This theory happened to be spectacularly successful in explaining several of the observed properties of stars.

Suggested Reading-

A. S. Eddington, 1920, ***The Internal Constitution of Stars***, Cambridge Science Classics

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Suggested Problems

1. (a) Show that the pressure differential between the two edges of a slab of thickness dx, on one side of which a flux of radiation, F, is incident is given by -

$$dP = F\kappa\rho dx/c\,,$$

irrespective of the angle of incidence. Here, κ is the mass absorption coefficient per gm, ρ is the density of the material in the slab, c is the velocity of light.

   (b) Using the above relation and the equation for hydrostatic pressure balance inside a star, obtain the following expression for Eddington luminosity -

$$L_{\rm Edd} = \frac{4\pi cGMm_H}{\sigma_{\rm Th}}\,,$$

where M, m_H and $\sigma_{\rm Th}$ are the mass of the star, mass of proton and the Thompson scattering cross section for the electron. Please carefully note the assumptions/approximations used to arrive at this relation.

   (c) Calculate the Eddington luminosity of the Sun, using the above relation and compare it with the observed luminosity of the Sun.

2. (a) Carefully retrace the steps to obtain the Mass-Luminosity relation in Eddington's theory of stars. Once again, note the assumptions/approximations. Do you find any difference between the assumptions made here and those made for Problem.1?

   (b) Using the Mass-Luminosity relation obtained above, calculate the ratio between the life-time of Sun and that of a star of mass $10\ M_\odot$.

   (c) We know that stars have masses in the range $10^{33} - 10^{35}$ gm. If the lifetime of the Sun happens to be $\sim 10^{10}$ years, estimate the range of stellar lifetimes.

   (d) The theoretical Mass - Luminosity obtained by Eddington, assuming ideal gas equation, fits the observed relation very well. Why does that happen?

   (e) The interior temperature of the Sun is 10^7 K. Find the density at which Hydrogen gas in the centre of the Sun begins to depart from ideal gas behaviour.

3. Subrahmanyam Chandrasekhar showed that a characteristic mass, given by

$$[M] = \left(\frac{hc}{G}\right)^{3/2} m_p^{-2}\,.$$

See if you can obtain this combination from purely dimensional arguments.

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