Lecture 10 : Quantum Statistics
Fermi-Dirac Statistics & the Degenerate Electron Gas

The dependence of the pressure of the gas on the density and temperature plays a crucial role in the evolution of the stars as they age. The evolution is very different for stars in which the gas obeys classical Boyles law, compared to stars in which the gas obeys the rules of quantum physics. As a prelude to a detailed discussion of the life history of stars in the subsequent lectures, in this lecture the Fermi-Dirac distribution is reviewed.

Fermi-Dirac statistics describes energy distribution of a non (or weakly) interacting gas of identical particles (called fermions, eg. electrons, protons, neutrons, neutrinos, quarks etc) that obey the Pauli exclusion principle. It is named after Enrico Fermi who derived it in 1926, and Paul Dirac who derived it independently a little later in the same year. The conduction electrons of the metals can be thought of as an ideal gas of electrons, and Fermi-Dirac statistics is successfully used to calculate metallic properties like the thermal conductivity, electrical conductivity etc. However, the most spectacular application of Fermi-Dirac statistics has been in the area of stellar remnants (white dwarfs, neutron stars). R. H. Fowler, in 1926 itself, proposed that the physics of white dwarfs can be explained best by considering them as an ideal non-relativistic gas of electrons obeying Fermi-Dirac statistics. Soon afterwards, Subrahmanyan Chandrasekhar obtained the maximum mass of a white dwarf using this concept. Independently, Landau obtained the value of maximum mass not only for white dwarfs but also for the 'neutron stars' (almost on the eve of the discovery of neutrons!).

Basic Texts -

1. C. B. Kittel, 2012 (8th ed.), ***Introduction to Solid State Physics***, Wiley
2. P. B. Pal, 2007, ***An Introductory Course of Statistical Mechanics***, Narosa Publishing House
3. R. K. Pathria & P. Beale, 2021, ***Statistical Mechanics***, Elsevier

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Prepared by Dr. Sushan Konar : ***sushan.konar@gmail.com***

Astronomy & Astrophysics : An Introductory Survey

A lecture series by Prof. G. Srinivasan

A 'Golden Jubilee Celebration' Event of the Astronomical Society of India

Lecture 10 : Quantum Statistics

[Supplementary Material : Dr. Sushan Konar]

Suggested Problems

1. At the centre of the Sun, the density of the ionised Hydrogen gas is $\sim 150 \mathrm{gm.cm}^{-3}$and the temperature is $\sim 10^7$K. Show that the particles would obey Fermi-Dirac statistics at the centre of the Sun.
2. The Fermi energy (E_F) for potassium is 2.1eV. Calculate the velocity of the electrons at the Fermi level.
3. In a solid, consider the energy level lying 0.1eV below the Fermi level. What is the probability of this level not being occupied by an electron at room temperature?
4. Show that the probability of finding an electron of energy ΔE above the Fermi level is same as the probability of not finding an electron at energy ΔE below the Fermi level.
5. Show that, at T=0, the chemical potential equals the Fermi energy.
6. Find the equation of state for an ideal Fermi gas in the limit of high temperatures. Include the first correction due to quantum effects. Estimate, for each of the ideal gapes, the temperature below which quantum effects would be important - electrons, atomic hydrogen, atomic Helium.
7. Consider a gas of free spin-half fermions in two dimensions. The gas is contained within an area A.
   (a) What is the density of one-particle levels in k-space?
   (b) How does the Fermi energy E_F depend upon the density N/A?
   (c) Use $\Sigma_r < n_r >= N$ to show that $\mu + k_B T \ln(1 + e^{-\mu/k_B T}) = E_F$.

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