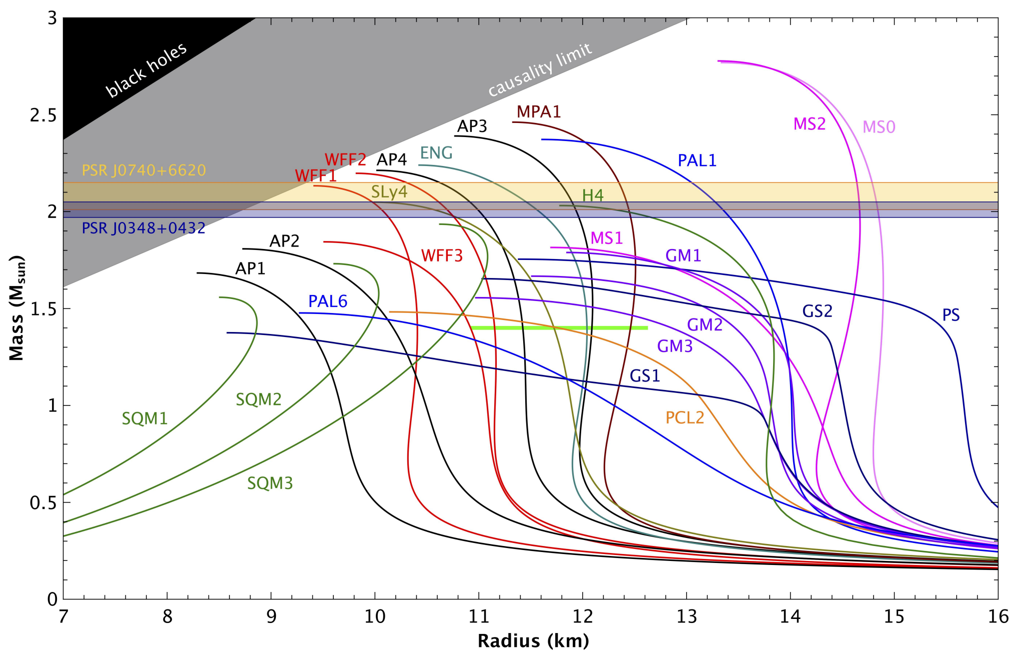
Astronomy & Astrophysics : An Introductory Survey
A lecture series by Prof. G. Srinivasan
A 'Golden Jubilee Celebration' Event of the Astronomical Society of India

Lecture 21 : Maximum mass of Neutron Stars

It is but natural to ask what Robert J. Oppenheimer, the Gita-quoting leader of the 'Manhattan Project and one of the most charismatic physicist involved in the Allied Forces' war effort, might have been doing prior to his Los Alamos stint. Not many realise that he had actually been delving deep inside neutron stars, trying to find their maximum mass in collaboration with Russian-Canadian physicist George Michael Volkoff.

By 1934, Chandrasekhar had clearly established that white dwarfs cannot exist with mass greater than a limiting mass, now referred to as the Chandrasekhar Limit. Soon after the 'neutron' was discovered, physicists made stars entirely of neutrons, and it appeared for a while that very massive stars will find their ultimate peace as neutron stars. But this hope was dashed in 1938 when Oppenheimer and his student Volkoff proved that there is a limiting mass for neutron stars also, analogous to the Chandrasekhar limit. This once again raised the question "What will be the fate of very massive stars?" In a historic paper published in 1939, Oppenheimer and Snyder proved emphatically that massive stars will end their lives as Black Holes, predicted by Einstein's General Theory of Relativity. But Einstein summarily rejected the notion of Black Holes. This lecture is devoted to these dramatic developments.



Great advances have been made in our understanding of nuclear interactions since the time of Oppenheimer. Yet, serious uncertainties regarding the true nature of dense matter remain. This is best illustrated by the large number of equation of state (EOS), used to describe the interior of neutron stars, that exist today. For each EOS (named in the adjacent figure) the mass-radius relation for neutron stars is indicated by its related curve.

The horizontal lines are 68.3% confidence limits for two of the most massive neutron stars. If a particular EOS predicts a maximum mass smaller than the largest measured neutron star mass then that EOS can not be a representative of the neutron star matter. Clearly, the recent (2017) detection of J0952-0607 with a mass of $\sim 2.35\ M_\odot$ (not shown in the figure) excluding all but a very few select EOS is one of the most interesting developments in the study of EOS.

[Figure Credit : Norbert Wex. Taken from the homepage of Paolo Freire.]

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[Supplementary Material : Dr. Sushan Konar]

Suggested Problems

1. **Mass - Radius Relation : White Dwarf**

   Write a numerical program (in your favourite coding language) to integrate the equation of hydrostatic pressure balance (in conjunction with the equation of state and the mass-radius relation) shown below to obtain the density profile, i.e, density as a function of radius, $\rho(r)$, inside a White Dwarf. Use electron degeneracy pressure for the equation of state.

$$\frac{dP}{dr} = -\frac{GM(r)\rho(r)}{r^2}\,, \quad \text{hydrostatic pressure balance} \tag{1}$$

$$\frac{dM}{dr} = 4\pi r^2 \rho\,, \quad \text{mass-radius relation} \tag{2}$$

$$P = P(\rho)\,, \quad \text{equation of state} \tag{3}$$

   You should integrate from the centre, $r = 0$, with an assumed central density, ρ_c, to the surface to obtain the total mass and the radius of the star. The surface is defined as that radius (R) at which the pressure goes to zero. Numerically, that would be equivalent to setting $P(R) = \epsilon P_c$, where P_c is the pressure at the centre and ϵ is a very small number (that your numerical program is capable of handling).

   Find the mass and radius for a range of central densities and generate the Mass-Radius curve to see that the curve asymptotically goes to zero radius as $M \to M_{\rm Ch}$ with increasing values of ρ_c.

2. Repeat the above exercise by replacing the equation for hydrostatic pressure balance by its relativistic equivalent, i.e., the TOV equation shown below -

$$\frac{dP(r)}{dr} = -\frac{G\left(M(r) + \frac{4\pi r^3 P(r)}{c^2}\right)\left(\rho(r) + \frac{P(r)}{c^2}\right)}{r^2\left(1 - \frac{2GM(r)}{r^2 c^2}\right)}\,. \tag{4}$$

   Convince yourself that the effect of general relativity is minimal for White Dwarfs.

3. The reason behind the stability of the neutrons inside a neutron star has been explained in the lecture. Find the minimum density at which neutrons are stable against the following decay mode, in the rest frame of the neutron. Assume that the fraction of proton in the nuclear matter considered is 10%.

$$n \to p + e + \bar{\nu}\,. \tag{5}$$

4. **Mass - Radius Relation : Neutron Star**

   Repeat the exercise of Problem 2, replacing electron degeneracy pressure by neutron degeneracy pressure. The Mass-Radius relation obtained should turn inwards at a particular point. Check that it matches with the maximum neutron star mass obtained by Oppenheimer & Volkoff, i.e., at 0.7 $M_\odot$.

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