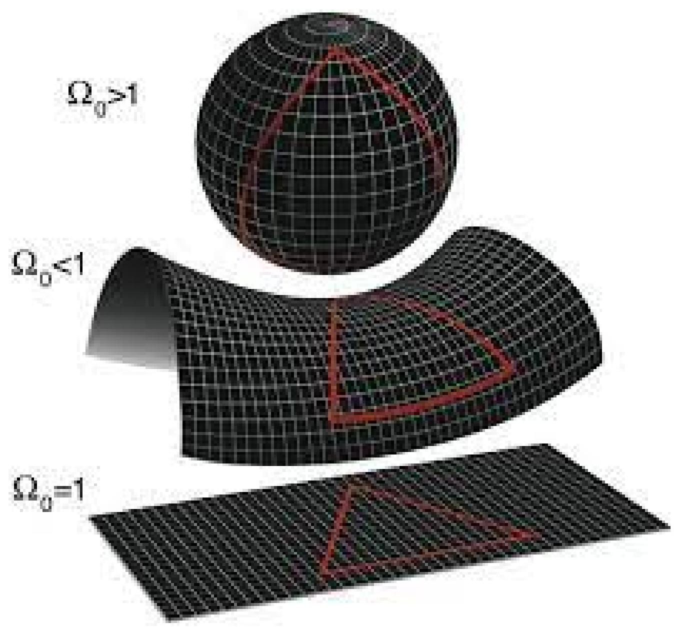
Lecture 38 : The Big Bang

Around 1970, the following embarrassing questions were staring at us. (1) Why is our universe expanding? (2) How can our universe be so homogeneous and isotropic? (3) Why is the present density of the universe so close to the 'critical density' predicted by General Relativity the density which would imply that the 'Geometry' of the universe must be Euclidean? In 1981, Alan Guth made a great discovery that might have answered all these questions in one go. In this lecture, first the three questions mentioned above are elaborated upon. Then an outline of Guth's theory of the exponential expansion of the universe, and how it 'solves' the three mysteries mentioned above is given



WMAP has determined the geometry of the universe to be nearly flat. However, under Big Bang cosmology, curvature grows with time. A universe as flat as we see it today would require an extreme fine-tuning of conditions in the past, in non-inflationary cosmology. For example, it would require the density at the Planck time (within 10^{-43} seconds of the Big Bang) to be within 1 part in 10^{57} of the critical density.

20 January 2023

Astronomy & Astrophysics : An Introductory Survey

A lecture series by Prof. G. Srinivasan

A 'Golden Jubilee Celebration' Event of the Astronomical Society of India

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Suggested Problems

1. ***Horizon :*** The success of the Big Bang predictions for the abundances of the light elements suggests that the universe was already in thermal equilibrium at one second after the Big Bang. At this time, the region that later evolves to become the observed universe was, in the context of the conventional (non-inflationary) cosmological model, many horizon distances across. Try to estimate how many. You may assume that the universe is flat, that it was radiation-dominated for $t \lesssim 50,000$ yr, and for this crude estimate you can also assume that it has been matter-dominated for all $t \gtrsim 50,000$ yr, and that $a(t)\,T(t)$ to be constant for the whole period from 1 second to the present.

2. ***Flatness :*** Although we now know that $\Omega_0 = 1$ to an accuracy of about half a percent, let us pretend that the value of Ω today to be 0.1. It nonetheless follows that at 10^{-37} second after the Big Bang (about the time of the grand unified theory phase transition), Ω must have been extraordinarily close to one. Writing $\Omega = 1 - \delta$, estimate the value of δ at $t = 10^{-37}$ sec (using the standard cosmological model). You may again use any of the approximations mentioned in the previous problem.

3. ***Monopole :*** The Grand Unified Theories (GUT) imply that the existence of magnetic monopoles, which form as "topological defects" in the configuration of the Higgs fields, are responsible for breaking the grand unified symmetry to the SU(3)×SU(2)×U(1) symmetry of the standard model of particle physics. If GUT theories and the conventional (non-inflationary) cosmological model were both correct, then far too many magnetic monopoles would have been produced in the Big Bang.

   At very high temperatures the Higgs fields oscillate rapidly, and therefore average out to zero. As the temperature falls, the system undergoes a phase transition at the critical temperature $k_B T_c = 10^{16}$ GeV. At this phase transition, the Higgs fields acquire nonzero expectation values, and the grand unified symmetry is spontaneously broken. The monopoles are twists in the Higgs field expectation values, so the monopoles form at this phase transition.

   Now, each monopole is expected to have a mass $M_M c^2 = 10^{18}$ GeV. According to an estimate first proposed by T. W. B. Kibble, the number density n_M of monopoles formed at the phase transition is of the order $n_M \sim \xi^{-3}$, where ξ is the correlation length of the field, defined roughly as the maximum distance over which the field at one point in space is correlated with the field at another point in space. The correlation length can not be larger than the physical horizon distance at the time of the phase transition, and it is believed to be comparable to this upper limit. An upper limit on ξ is a lower limit on n_M. Therefore, there must be at least an order one monopole per horizon-sized volume.

   Assume that the particles of the GUT form a thermal gas of blackbody radiation, with energy density $u = g_{GUT}\pi^2(k_B T)^4/30(\hbar c)^3$ with $g_{GUT} \sim 200$. Further assume that the universe is flat and radiation-dominated from its beginning to the time of the GUT phase transition t_{GUT}.

   (a) Under the assumptions described above, at what time t_{GUT} does the phase transition occur?

   (b) Setting ξ equal to the horizon distance, estimate the number density n_M of magnetic monopoles just after the phase transition.

   (c) Calculate the ratio n_M/n_γ of the number of monopoles to the number of photons immediately after the phase transition. Assume that the temperature after the phase transition is still approximately Tc.

   (d) For topological reasons monopoles cannot disappear, but they form with an equal number of monopoles and anti-monopoles, where the anti-monopoles correspond to twists in the Higgs field in the opposite sense. Monopoles and anti-monopoles can annihilate each other, but estimates of the rate for this process show that it is negligible. Thus, in the context of the conventional (non-inflationary) hot Big Bang model, the ratio of monopoles to photons would be about the same today as it was just after the phase transition. Use this assumption to estimate the contribution that these monopoles would make to the value of Ω today.

20 January 2023